Extremely difficult instances of M by N (draft 1)

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1 Introduction

The "M by N" family of concepts has a long and storied history. They ultimately originate from the Single concept, which in the language of "M by N" could be described as 1x1, defined in 1987 by Eric Brosius[1]. Following the introduction of Single, the similar 3x3 concept was defined in 1991 by Sue Curtis[2] (as well as related instances like 4x4), and then the 3x1 concept in 1993 also by Sue Curtis[3] (as well as related instances like 1x2). Finally, usages such as 4x0 and 0x4 are now also seen, which to my understanding is largely pioneered by Atsushi Takeuchi.

In this paper, I will describe a number of ways to make this concept extremely difficult to dance. With a few exceptions, I have seen essentially none of these called in practice. However, in my opinion, they are very fun and interesting to think about from the comfort of your own home, with the freedom to take as much time as you want to figure them out.

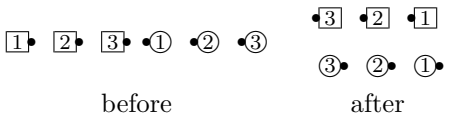
2 One-faced interleaved 1x3

This discussion is likely well-trodden ground for many readers in the audience of this paper, so I will summarize as briefly as possible.

Typically, there are only two ways that the groups in M by N can be positioned relative to each other. In Wheel and Deal, for example, the pairings are between the people in each couple; whereas in Switch the Wave, the pairings are between the people once removed from each other. I will use the word "together" when the groups are couples or tandems, and "interleaved" when the groups are once removed couples or once removed tandems.

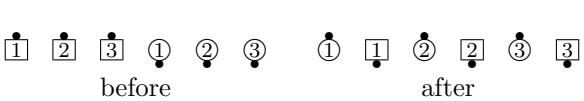
The groups can independently start either together or interleaved, and end either together or interleaved.

3x3 Recoil



(start together, end together)

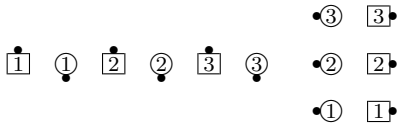
3x3 Cross Roll



(start together, end interleaved)

[1] https://challengedance.org/single/single.html
[2] https://challengedance.org/3by3/3by3.html
[3] https://challengedance.org/3by1/3by1.html

3x3 Nicely



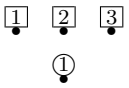
(start interleaved, end together)

3x3 Ah So and Recycle

(start interleaved, end interleaved)

On 1x3, if the groups start facing the same way, the rule is 1 leader and 3 trailers, or 1 belle and 3 beaus. This works perfectly well for calls that start together.

before 1x3 Turn and Deal



after

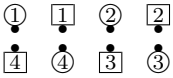
However, on calls that start interleaved, the "1" cannot be the belle-most dancer, because then the groups would not be interleaved. Hence, the "1" must be the *second*-belle-most dancer, and the phantoms must be inserted at distance 2 on either side of them (instead of distance 1). This is the same result one would get by starting with the 3x3 version and eliminating the outer dancers of the belle-most group.

before 1x3 Switch the Line

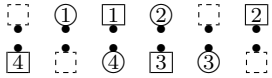
insert phantoms (3x3 version)

after

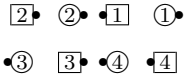
Of course, this is relevant for the infamous 1x3 Load the Boat.



before 1x3 Load the Boat



insert phantoms (3x3 version)

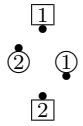


after

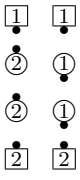
3 Mismatched pairings

The pairings on M by N are almost invariably the same "nature" for everyone – it's either all the couples, or all the tandems, or all the once removed couples, or all the once removed tandems, or what have you.

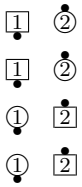
A relatively tame example where this is not true, which I first saw when Max Murin called it at a dance weekend in 2025, is Single Triple Cross (which works from multiple formations!).



before Single Triple Cross



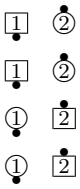
virtual formation



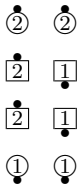
virtual call

after Single Triple Cross

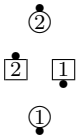
before Single Triple Cross



virtual formation

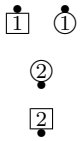


virtual call

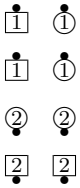


after Single Triple Cross

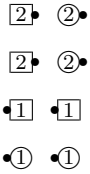
A more exotic example, which I learned from V Kehoe, is Single Countershake from the formation below.



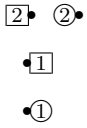
before Single Countershake



virtual formation



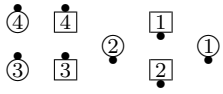
virtual call



after Single Countershake

(This observation can lead to some truly alarming applications.)

before Once Removed Single Countershake

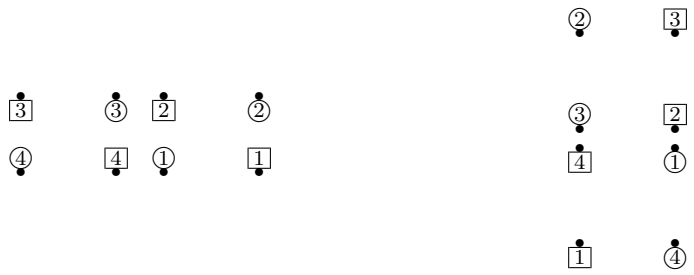


after

In general, this kind of application can occur with any call that starts from once-faced boxes and can be done Single Single. Other examples are Turn Away, Hang a Right, and Polly Wally.

4 Diagonal pairings

The example below was called by Noriko Takahashi at a dance weekend in 2026.



before 3x3 Rip the Line | after

Actually, the desired ending formation was a 4x6 by caller fiat, but I think the above result is how the call "should" work (whatever that means). Note that the phantoms are standing on top of each other.

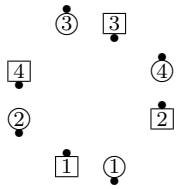
I learned of the similar example below from Della Hendrickson, which starts in overlapped diagonals instead of ending there.

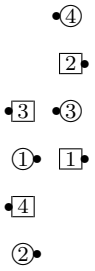
before 3x3 Magic Transfer | after

Taking it a step further, since the groups are facing opposite directions, the center spot can have a real person (standing on top of a phantom's head).

before 3x3 Magic Transfer | after

Taking it yet another step further, why not have both "cheeses" as real people?

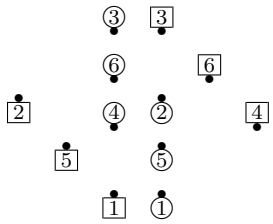


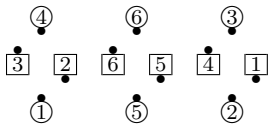


before Common Spot 3x6 Matrix, 3x3 Magic Transfer

after

Combining diagonal pairings with axis-aligned pairings can result in some extremely confusing questions about what matrix spots people should be in.



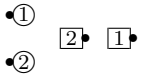


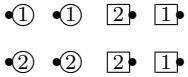
before 3x3 Flip Your Lid??

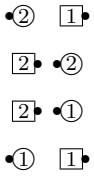
after??

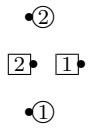
5 Mismatched *and* diagonal pairings

Combining the phenomena in the previous two sections can result in some even more terrifying applications. In both of these examples, the dancers start with mismatched axis-aligned pairings, and end with some diagonal and some axis-aligned pairings.







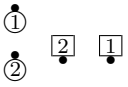


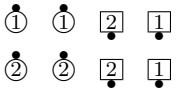
before Single Stable Counter Clear Out

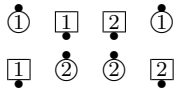
virtual formation

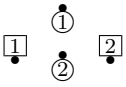
virtual call

after









before Single Stable Outroll Clear Out

virtual formation

virtual call

after

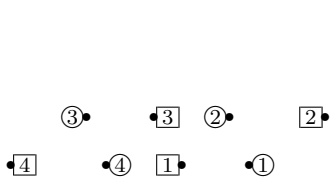
6 Phantoms

Consider the call "2x4 Matrix Perk Up" from the given formation.

before 2x4 Matrix Perk Up

after

Aside: why call it by this name? It is the thing that 4 people would do on a Phantom Waves Perk Up if they were in the middle, or that each side would do on a Split Phantom Waves Perk Up. As far as I know, there isn't a convenient name for this in square dancing. For example, what if you wanted to use a phantom call as an input to Checkpoint?



before Checkpoint 2x4 Matrix
Wind the Bobbin by Z Axle

after

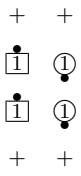
Regardless – the motion of 2x4 Matrix Perk Up satisfies all the conditions in the definition of the M by N concept, so there is no reason we shouldn't be able to apply it. Note that *the phantoms do not satisfy the conditions!* The missing people would be turning opposite directions on the All 8 Circulate. But the M by N concept does not "see" the phantoms; all it sees is a dance motion, and it doesn't look inside the call to see how that dance motion was derived. This is why Single Perk Up is not legal from a mini-wave box, but Single 2x4 Matrix Perk Up is legal from a facing skew (with the leaders of the mini-wave box removed).

before 4x4, 2x4 Matrix Perk Up

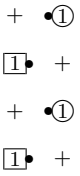
after

This kind of usage can produce some exceedingly computationally intensive calls. In order to evaluate a call of this form, the dancers must first imagine the full version of the call, and pick out which spots live people can be placed in such that the resulting M by N is compatible with the real starting formation. Here are a number of examples.

before Single, 2x4 Matrix,
Shadow the Column



unique valid
instantiation

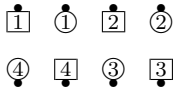


virtual call

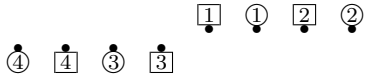
after

For a short bit of commentary on how to compute the above example: clearly we must start in right-handed columns. In order for Single to produce people who are lined up, the real people must either be the #1 and #4 or the #2 and #3. But the first 3 in the column all turn left while the #4 turns right, so the #4 can't be one of the real people. It therefore must be the #2 and #3. (Also, the #1 turns 5/4 to the left over the course of the call, while the others only turn 3/4.)

Interestingly, the call "Single 2x4 Matrix [Shadow the Column and Extend]" is choreographically equivalent to Snake.



before 4x4, 2x4 Matrix Roll Em



after

before Single, 2x4 Matrix,
1/4 Stable Plan Ahead

after

before Single, 2x4 Matrix Triple Play
(acting as #2 and #3)

after

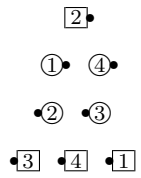
before Single, 2x4 Matrix Triple Play
(acting as #1 and #3)

after

7 "You know what I mean"

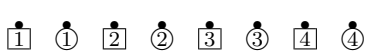
Some usages of this concept do not fit any formal definition, but it is so perfectly clear what the caller wants that nobody has any problem doing it.

At a dance weekend in 2024, I called "3x2x2x1 Drift Apart" from the following asymmetric formation:

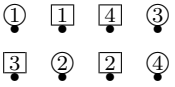


The dancers did it immediately without complaints. Of course, the same sequence had a Convert the Pentagon, so they might have just already been expecting nonsense. (The next call was Lines Reverse Flip the 3x1 Triangle Thru restoring symmetry, in case you're wondering how to get out of that mess.)

Asymmetrics are rife with opportunites for nonsense like this.



before 1x2x3x2 Wheel and Deal



after

8 An amusing question

From here:

would you do a 3x5 Cross Roll like this…

or like this?